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Supersymmetric world-volume action for non-BPS D-branes

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Abstract: We construct the world-volume action for non-BPS D-branes in type II string theories. This action is invariant under all the unbroken supersymmetries in the bulk, but these symmetries are realised as spontaneously broken symmetries in the world-volume theory. Coupling of this action to background supergravity fields is straightforward. We also discuss the fate of the U(1) gauge field on the D-brane world-volume after tachyon condensation.

Keywords: D-branes, Superstrings and Heterotic Strings.

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Contents

1. Introduction and summary

During the last year it has been realised that type IIA (IIB) string theory admits unstable non-BPS D-branes of odd (even) dimensions [1, 2, 3]. These D-branes can give rise to stable non-BPS D-branes after we mod out the original type II string theory by some discrete symmetry group [4, 5, 6, 1, 7, 2, 8]. Since the non-BPS D-brane is not invariant under any of the space-time supersymmetry transformations, the spectrum on the world-volume of these D-branes does not have Bose-Fermi degeneracy in general, although in some special cases the spectrum can develop accidental Bose-Fermi degeneracy [9].

However, although there is no manifest supersymmetry in the world-volume theory, we still expect the world-volume theory to be supersymmetric, with the supersymmetry realised as a spontaneously broken symmetry.[1] The supersymmetry generators of the bulk theory, acting on the non-BPS D-brane, produce fermion zero modes which can be identified as the goldstinoes associated with the spontaneously broken supersymmetry. Indeed, even for an ordinary BPS D-brane, which is invariant under half of the space-time supersymmetry of the bulk theory [10], the world-volume theory possesses all the supersymmetries of the bulk theory, with half of them realised as unbroken symmetries, and the other half realised as spontaneously broken symmetries.

[1] I wish to thank O. Aharony, B. Kol and Y. Oz for discussion on this point.

In this paper we shall construct the supersymmetric generalization of the Dirac-Born-Infeld (DBI) action describing the dynamics of light modes on the world-volume of a non-BPS D-brane. In section 2 we focus on the world-volume theory of the massless fields of non-BPS D-brane of type II string theory in Minkowski space-time. As we shall see, this can be constructed with almost no work, using the κ-symmetric action for a BPS D-brane derived in ref. [11]–[16]. In the convention of refs. [14, 15, 16] the κ symmetric action for a BPS D-brane has two parts, $-$ the supersymmetric DBI action and the Wess-Zumino term. Each of these terms is separately invariant under the full set of space-time supersymmetry transformations, but only the combined system has κ gauge invariance. To begin with, the action has double the number of fermionic degrees of freedom compared to the number of physical degrees of freedom on a BPS D-brane, but fixing the κ gauge symmetry removes half of these degrees of freedom. We show that the world-volume action describing the non-BPS D-brane is given by just the supersymmetric DBI part of the κ symmetric action describing a BPS D-brane. By construction the action is invariant under all the space-time supersymmetries. But it does not have the κ gauge symmetry, and hence the number of physical massless fermionic degrees of freedom is exactly double of that on a BPS D-brane. This is precisely the case for a non-BPS D-brane. The open strings living on a non-BPS D-brane have an extra sector in which the GSO projection is reversed, and hence there are double the number of massless fermionic degrees of freedom on a non-BPS D-brane compared to that on a BPS D-brane [5, 6, 1].

If we want to construct the world-volume action of a non-BPS D-brane on an orbifold or an orientifold of type II string theory, then typically the action is obtained from the action described above by removing the degrees of freedom which are projected out under the orientifold or the orbifold operation. However, in some cases we may get extra light degrees of freedom after the orbifold/orientifold operation. Thus for example a type I 0-brane acquires extra fermionic zero modes from open string stretched between the 0-brane and the 9-brane [6, 1]; these degrees of freedom are not present in the 0-brane of type IIB string theory. Another example involves non-BPS D-brane of type II string theory on a K3 orbifold. In this case at special points in the moduli space of K3 the world-volume theory of the brane may contain extra massless scalar fields which are not present in the non-BPS D-branes of type II string theory in the ten dimensional Minkowski space-time. In section 3 we discuss inclusion of these fields in the world-volume action maintaining supersymmetry of the world-volume action.

In section 4 we discuss the effect of including the tachyon in the world-volume action of the non-BPS D-brane of type II string theory in Minkowski space-time. In this case the tachyon mass^2 is of the order of the string scale, and hence there is no systematic way of constructing the effective world-volume action involving the tachyon. But we can still write down the general form of the action that maintains supersymmetry. From general arguments one expects that at the minimum of the tachyon potential the configuration is indistinguishible from the vacuum [17], and hence the U(1) gauge field and the other degrees of freedom on the D-brane world-volume should disappear; but exactly how this happens has not been completely understood [18, 7, 19]. We show that if we ignore terms involving derivatives of the gauge field strength and the acceleration of the brane, then at the minimum of the tachyon potential the world-volume action vanishes identically. Thus the gauge field now acts as a lagrange multiplier field which imposes the constraint that the U(1) gauge current must vanish identically. This would explain the disappearance of the U(1) gauge field at the tachyonic ground state, but in order to reach a definitive conclusion we need to study the effect of the terms in the world-volume action involving higher derivatives which were ignored in our analysis.

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2. World-volume action of massless fields for non-BPS D-branes in type II string theory

We shall begin with type IIA string theory, which admits BPS D-branes of even dimension and non-BPS D-branes of odd dimension. Our focus of attention will be a non-BPS Dp-brane with p odd, and we shall attempt to construct the world-volume action involving the massless fields on the brane by integrating out all the massive modes, including the tachyon.[2] Throughout this section we shall consider the background space-time to be ten dimensional Minkowski space with no background fields, but as we shall point out at the end of the section, coupling to background supergravity fields is straightforward. Let σ^μ $(0 \leq \mu \leq p)$ denote the coordinates on the world-volume of the D-brane. The open strings living on such a D-brane has two Chan Paton (CP) sectors, the sector labelled by the 2×2 identity matrix I, and the sector labeled by the Pauli matrix σ_1 [1]. The massless dynamical degrees of freedom on the D-brane world-volume are a set of 10 bosonic coordinate fields $X^M(\sigma)$ $(0 \leq M \leq 9)$, a U(1) gauge field A_μ, and a 32 component fermionic field θ which transforms as a Majorana spinor under the space-time Lorentz group SO(9,1), but is a world-volume scalar. The field θ can be regarded as the sum of a left-handed Majorana-Weyl fermion θ_L and a right-handed Majorana-Weyl fermion θ_R. Of these all the fields except θ_L come from the CP sector I, and θ_L comes from the sector σ_1.[3] For comparison let us note that the spectrum of massless fields coming from the sector I is identical to that on the world-volume of a BPS Dp brane of type IIB string theory. Thus for the BPS Dp-brane the fermionic field transforms in the Majorana-Weyl representation rather than in a Majorana representation.

[2] Throughout this paper we shall be working at the open string tree level, so integrating out the massive modes amounts to eliminating them using their equations of motion.

[3] Of course by changing our convention we could have the right-handed component of θ come from sector σ_1 and the left-handed component come from the sector I.

The world-volume action of the non-BPS D-brane involving these fields must satisfy the following criteria:

1. It must be invariant under all the global supersymmetries of type IIA string theory in the bulk. The supersymmetry transformation parameter is a Majorana spinor ϵ of SO(9,1).

2. If we set $\theta_L = 0$, then we are left with only the fields originating in the identity sector. Except for an overall normalization factor representing the difference in the tension of a non-BPS and a BPS D-brane, the resulting effective action must agree with that on a BPS Dp-brane of type IIB string theory. This follows from the observation that the rules for computing the open string amplitudes on a non-BPS D-brane are identical to that on the BPS D-brane except for the presence of the Chan-Paton factors [1], and a CP factor I only contributes an overall normalization factor to the amplitude.

We shall now use these guidelines to construct the world-volume action on the non-BPS Dp-brane of type IIA string theory. Let us define:

$$\Pi_\mu^M = \partial_\mu X^M - \bar\theta\Gamma^M\partial_\mu\theta\,, \tag{2.1}$$

$$\mathcal{G}_{\mu\nu} = \eta_{MN}\Pi_\mu^M\Pi_\nu^N\,, \tag{2.2}$$

and

$$\mathcal{F}_{\mu\nu} = F_{\mu\nu} - \left[\bar\theta\Gamma_{11}\Gamma_M\partial_\mu\theta\left(\partial_\nu X^M - \frac{1}{2}\bar\theta\Gamma^M\partial_\nu\theta\right) - (\mu\leftrightarrow\nu)\right], \tag{2.3}$$

where Γ^M denote the ten dimensional gamma matrices, Γ_{11} is the product of all the gamma matrices, η_{MN} is the ten dimensional Minkowski metric with signature $(-1,1,\ldots 1)$, and

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu\,. \tag{2.4}$$

We now claim that the following action satisfies the two conditions listed above:

$$S = -C\int d^{p+1}\sigma\sqrt{-\det(\mathcal{G}_{\mu\nu}+\mathcal{F}_{\mu\nu})}\,, \tag{2.5}$$

where C is a constant equal to the Dp-brane tension.

First let us check that it has the required amount of supersymmetry. For this note that this action has the same structure as the first term of the κ symmetric action of a BPS D-brane of type IIA string theory as discussed in ref. [16], except that p is odd instead of even in the present case. As shown in [16], both $\mathcal{G}_{\mu\nu}$ and $\mathcal{F}_{\mu\nu}$ are invariant under the supersymmetry transformations:

$$\begin{aligned}
\delta_\epsilon\theta &= \epsilon,\\
\delta_\epsilon X^M &= \bar\epsilon\Gamma^M\theta,\\
\delta_\epsilon A_\mu &= \bar\epsilon\Gamma_{11}\Gamma_M\theta\partial_\mu X^M - \frac{1}{6}\left(\bar\epsilon\Gamma_{11}\Gamma_M\theta\bar\theta\Gamma^M\partial_\mu\theta + \bar\epsilon\Gamma_M\theta\bar\theta\Gamma_{11}\Gamma^M\partial_\mu\theta\right),
\end{aligned} \tag{2.6}$$

where the supersymmetry transformation parameter ϵ is a Majorana spinor of SO(9,1) Lorentz group. Since $\mathcal{G}_{\mu\nu}$ and $\mathcal{F}_{\mu\nu}$ are invariant under the supersymmetry transformation, the action S defined in (2.5) is also invariant under this transformation.

In order to check that this reproduces the world-volume action of the BPS Dp-brane of type IIB string theory when we set the fermionic field θ_L coming from the CP sector σ_1 to zero, let us define θ_L and θ_R through the relations:

$$\theta = \theta_L + \theta_R\,, \qquad \bar\theta_L \Gamma_{11} = \bar\theta_L\,, \qquad \bar\theta_R \Gamma_{11} = -\bar\theta_R\,. \tag{2.7}$$

As discussed earlier, we can take θ_R to originate in the identity sector and θ_L to originate in the σ_1 sector. Setting θ_L to zero gives us

$$\begin{aligned} \mathcal{G}_{\mu\nu} + \mathcal{F}_{\mu\nu} = {} & \eta_{MN}\partial_\mu X^M \partial_\nu X^N + F_{\mu\nu} - \\ & - 2\bar\theta_R \Gamma_M \partial_\mu X^M \partial_\nu \theta_R + \left(\bar\theta_R \Gamma^M \partial_\mu \theta_R\right)\left(\bar\theta_R \Gamma_M \partial_\nu \theta_R\right). \end{aligned} \tag{2.8}$$

The action obtained by substituting this into eq. (2.5) agrees with the world-volume action of a BPS Dp-brane of type IIB string theory after fixing the κ gauge symmetry [16]. The θ_R appearing in (2.8) has to be identified to the field λ of ref. [16].

Besides supersymmetry, the action (2.5) also has several other symmetries. They include space-time translation symmetry,

$$\delta_\xi X^M = \xi^M\,, \qquad \delta_\xi \theta = 0\,, \qquad \delta_\xi A_\mu = 0\,, \tag{2.9}$$

and the SO(9,1) Lorenz symmetry,

$$\delta_\Lambda X^M = \Lambda^M_{\ N} X^N, \qquad \delta_\Lambda \theta = R(\Lambda)\theta, \qquad \delta_\Lambda A_\mu = 0\,. \tag{2.10}$$

Here $\Lambda^M_{\ N}$ denotes an infinitesimal element of the so(9,1) algebra, and $R(\Lambda)$ is the Majorana spinor representation of Λ. Finally, the action is manifestly invariant under the reparametrization of the world-volume coordinate σ:

$$\sigma^\mu \to f^\mu(\sigma)\,, \tag{2.11}$$

for some set of functions $\{f^\mu(\sigma)\}$. X^μ and θ transform as world-volume scalars and A_μ transforms as a world-volume vector under this transformation. (2.11) represents a gauge symmetry of the action. A convenient choice of gauge is the static gauge:

$$\sigma^\mu = X^\mu \qquad \text{for } 0 \le \mu \le p\,. \tag{2.12}$$

If we define

$$\phi^i = X^i \qquad \text{for } p+1 \le i \le 9\,, \tag{2.13}$$

then $\mathcal{G}_{\mu\nu} + \mathcal{F}_{\mu\nu}$ can be rewritten in the static gauge as:

$$\begin{aligned}\mathcal{G}_{\mu\nu} + \mathcal{F}_{\mu\nu} = &\ \eta_{\mu\nu} + \delta_{ij}\partial_\mu\phi^i\partial_\nu\phi^j + F_{\mu\nu} - \\ &- 2\bar\theta_L\left(\Gamma_\nu + \Gamma_i\partial_\nu\phi^i\right)\partial_\mu\theta_L - 2\bar\theta_R\left(\Gamma_\mu + \Gamma_i\partial_\mu\phi^i\right)\partial_\nu\theta_R + \\ &+ \left(\bar\theta_L\Gamma^M\partial_\mu\theta_L\right)\left(\bar\theta_L\Gamma_M\partial_\nu\theta_L + \bar\theta_R\Gamma_M\partial_\nu\theta_R\right) + \\ &+ \left(\bar\theta_R\Gamma^M\partial_\nu\theta_R\right)\left(\bar\theta_L\Gamma_M\partial_\mu\theta_L + \bar\theta_R\Gamma_M\partial_\mu\theta_R\right). \end{aligned} \tag{2.14}$$

If we set $\theta_L = 0$, and rename θ_R as λ, the action agrees with the action of BPS Dp-brane of type IIB string theory in the static gauge, as given in ref. [16].

The gauge fixed action is no longer invariant under the symmetry transformations (2.6), (2.9) and (2.10). Instead, each of these symmetry transformations must be accompanied by a compensating gauge transformation which brings us back to the gauge $X^\mu = \sigma^\mu$. Thus we must define the new symmetry transformations $\widehat{\delta}$ as a combination of the old transformations δ and a world volume reparametrization, such that

$$\widehat{\delta}X^\mu(\sigma) = 0\,. \tag{2.15}$$

Thus the new transformation laws $\widehat{\delta}$ for any world-volume scalar field Φ (e.g. X^M or θ) will be given in terms of the old transformation laws δ as follows:

$$\widehat{\delta}_{\epsilon,\xi,\Lambda}\Phi = \delta_{\epsilon,\xi,\Lambda}\Phi - (\Delta^\mu_{\epsilon,\xi,\Lambda})\partial_\mu\Phi\,, \tag{2.16}$$

where

$$\Delta^\mu_\epsilon = \bar\epsilon\Gamma^\mu\theta\,, \qquad \Delta^\mu_\xi = \xi^\mu\,, \qquad \Delta^\mu_\Lambda = (\Lambda^\mu{}_\nu\sigma^\nu + \Lambda^\mu{}_i\phi^i)\,. \tag{2.17}$$

If we take $\Phi = X^\mu$, then it is easy to check using eqs. (2.6), (2.9), (2.10), (2.16) and (2.17) that eq. (2.15) is indeed satisfied. For world-volume vector field A_μ the right hand side of eq. (2.16) contains an extra term $-(\partial_\mu\Delta^\rho_{\epsilon,\xi,\Lambda})A_\rho$.

Let us now turn to non-BPS Dp-branes of type IIB string theory. In this case p is even. The bosonic massless fields again consist of the scalar fields $X^M(\sigma)$ and the gauge fields $A_\mu(\sigma)$, but now, instead of a fermionic field θ transforming as a Majorana spinor of SO(9,1), we have a pair of fermionic fields θ_1 and θ_2, each transforming in the right-handed Majorana-Weyl spinor representation of SO(9,1).[4] We define

$$\theta = \begin{pmatrix}\theta_1\\ \theta_2\end{pmatrix}\,, \tag{2.18}$$

[4]Since the fermion zero modes from CP factor σ_1 carry opposite GSO projection compared to those from CP factor I, one might wonder how we can get a pair of fermion fields of same SO(9,1) chirality. For this note that in the static gauge (in which the open string spectrum is computed) only an SO(p,1)×SO($9-p$) subgroup of the full SO(9,1) Lorentz group is realized as a manifest symmetry. Since p is even for type IIB string theory, neither SO($9-p$) nor SO($p,1$) has chiral spinor representation. As a result, both a left-handed and a right-handed Majorana-Weyl spinor of SO(9,1) will transform in the same representation of SO($p,1$)×SO($9-p$). Thus knowing the GSO projection rules we cannot determine the SO(9,1) chirality of the fermion fields. It is determined by requiring that these fermions represent the goldstino fields associated with spontaneously broken supersymmetry.

and let τ_3 denote the matrix $\begin{pmatrix} I & \\ & -I \end{pmatrix}$ acting on θ, where I denotes the identity matrix acting on θ_1 and θ_2. We also define

$$\begin{aligned}
\Pi^M_\mu &= \partial_\mu X^M - \bar{\theta}\Gamma^M\partial_\mu\theta\,, && (2.19)\\
\mathcal{G}_{\mu\nu} &= \eta_{MN}\Pi^M_\mu\Pi^N_\nu\,, && (2.20)\\
\mathcal{F}_{\mu\nu} &= F_{\mu\nu} - \left[\bar{\theta}\tau_3\Gamma_M\partial_\mu\theta\left(\partial_\nu X^M - \frac{1}{2}\bar{\theta}\Gamma^M\partial_\nu\theta\right) - (\mu\leftrightarrow\nu)\right], && (2.21)\\
S &= -C\int d^{p+1}\sigma\sqrt{-\det(\mathcal{G}_{\mu\nu}+\mathcal{F}_{\mu\nu})}\,, && (2.22)
\end{aligned}$$

where C is a constant equal to the Dp-brane tension. It is easy to check that $\mathcal{G}_{\mu\nu}$ and $\mathcal{F}_{\mu\nu}$, and hence S, is invariant under the supersymmetry transformation:

$$\begin{aligned}
\delta_\epsilon\theta &= \epsilon\,,\\
\delta_\epsilon X^M &= \bar{\epsilon}\Gamma^M\theta\,,\\
\delta_\epsilon A_\mu &= \bar{\epsilon}\tau_3\Gamma_M\theta\partial_\mu X^M - \frac{1}{6}\left(\bar{\epsilon}\tau_3\Gamma_M\theta\bar{\theta}\Gamma^M\partial_\mu\theta + \bar{\epsilon}\Gamma_M\theta\bar{\theta}\tau_3\Gamma^M\partial_\mu\theta\right), && (2.23)
\end{aligned}$$

where the supersymmetry transformation parameter ϵ is given by $\begin{pmatrix}\epsilon_1\\ \epsilon_2\end{pmatrix}$, with ϵ_1 and ϵ_2 both right-handed Majorana spinor of SO(9,1) Lorentz group.

If we set $\theta_1 = 0$ and identify θ_2 with λ, we recover the world-volume action of a BPS Dp-brane of type IIA string theory after κ gauge fixing, as given in ref. [16]. Thus the action (2.22) satisfies the required consistency conditions. As in the case of type IIA string theory, this action also has space-time translation symmetry and ten dimensional Lorentz invariance; these transformation laws are given by equations identical to (2.9) and (2.10). Finally it has world-volume reparametrization invariance, and using this we can go to a static gauge $X^\mu = \sigma^\mu$. The gauge fixed action has a form identical to that for type IIA non-BPS D-branes except that θ_L and θ_R in eq. (2.14) are replaced by θ_1 and θ_2 respectively. The various transformation laws get modified in the static gauge according to eqs. (2.16) and (2.17) as before.

We conclude this section by noting that the coupling of the D-brane world-volume theory discussed here to background supergravity fields can be carried out following the procedure given in ref. [14, 15]. Again for type IIA (IIB) string theory the action is obtained by keeping only the supersymmetric DBI term of the κ-symmetric action describing BPS D-brane of type IIA (IIB) string theory in a supergravity background, and taking p to be odd (even) instead of even (odd).

3. Inclusion of other light fields

The non-BPS Dp-branes of type II string theory are unstable due to the existence of a tachyonic mode on their world-volume [1, 2]. But quite often we can get stable non-BPS branes by taking certain orientifolds/orbifolds of type II string theory, if this operation projects out the tachyonic mode [5, 6, 1, 2, 8]. Typically, this will also project out a subset of the massless degrees of freedom on the D-brane world-volume, and the world-volume action of the resulting D-brane will be given by an appropriate truncation of the actions (2.5) or (2.22). But in some cases there are extra (nearly) massless degrees of freedom on the world-volume, which, if present, must be included in the world volume action. Thus we need to know how to couple these fields maintaining the ten dimensional super-Poincaré invariance and world-volume reparametrization invariance. We shall discuss two examples.

3.1 Light scalars on non-BPS D-brane on T^4/Z_2 near a critical radius

This system was discussed in detail in refs. [2, 8, 20, 9]. We begin with a non-BPS D-brane of type IIA/IIB string theory on T^4 with an odd number n of tangential directions of the D-brane along T^4, and mod out the resulting configuration by the Z_2 transformation $\mathcal{I}_4$ that reverses the sign of all four coordinates of the torus. At a generic point in the moduli space of the torus the massless degrees of freedom consist of a subset of the fields living on the D-brane before the orbifold projection. In order to simplify the action, we can do a partial gauge fixing by identifying n of the world-volume coordinates of the brane to the n coordinates of the torus along which the brane extends. In this case these n world-volume coordinates will be compact, and we can dimensionally reduce the original world-volume action by ignoring the dependence of all fields on these compact coordinates. This gives a $(p-n+1)$-dimensional world-volume theory. This dimensional reduction has been carried out in detail in [16]. In this dimensionally reduced action we then set to zero the fields which are odd under $\mathcal{I}_4$. We shall continue to denote by σ^μ the coordinates of this world-volume theory, although now μ runs over $(p-n+1)$ values.

An alternative procedure will be to make a series of n T-duality transformations which converts the Dp-brane to a D$(p-n)$-brane, with all the $(p-n)$ tangential directions of the brane lying along the non-compact directions [2, 8]. Since n is odd, this T-duality transforms $\mathcal{I}_4$ into $\mathcal{I}_4\cdot(-1)^{F_L}$ where $(-1)^{F_L}$ denotes the contribution to the space-time fermion number from the left-moving sector of the closed string world-sheet [23]. The world-volume action of the brane is then obtained by starting from the world-volume action of the non-BPS D$(p-n)$-brane discussed in the previous section, and setting to zero all the fields which are odd under $(-1)^{F_L}\cdot\mathcal{I}_4$. Both these procedures lead to the same result.

At certain critical values of the radii of the torus we can get one or more extra massless scalar fields [2, 8, 20].[5] For simplicity we shall take only one of the radii to be near the critical radius, so that we have only one nearly massless scalar field. Let us denote this field by χ. When all the other massless fields are set to zero, the low energy effective action for χ takes the following form in the static gauge:

$$\frac{1}{2}\int d^{p-n+1}\sigma(-\eta^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - V(\chi))\,, \tag{3.1}$$

where $V(\chi)$ is the tachyon potential constructed in [20]. In writing (3.1) we have ignored terms with more than two derivatives of χ.

[5]For a closely related example in type I string theory, see refs. [6, 21, 22].

We propose the following supersymmetric and reparametrization invariant coupling of the action (3.1) to other massless fields on the D-brane world-volume:

$$\int d^{p-n+1}\sigma\sqrt{-\det(\mathcal{G}+\mathcal{F})}\left(-\widetilde{\mathcal{G}}_S^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - V(\chi)\right), \tag{3.2}$$

where $\widetilde{\mathcal{G}}^{\mu\nu}$ denotes the matrix inverse of $(\mathcal{G}+\mathcal{F})$, and $\widetilde{\mathcal{G}}_S$ denotes the symmetric part of $\widetilde{\mathcal{G}}$. In writing (3.2) we have further ignored terms involving derivatives of $\mathcal{G}$ and $\mathcal{F}$. This action clearly satisfies the requirement of space-time supersymmetry and world-volume reparametrization invariance provided we take χ to be a world-volume scalar and inert under supersymmetry transformation:

$$\delta_\epsilon\chi = 0\,. \tag{3.3}$$

However, the requirement of supersymmetry alone does not fix the form of the action. For example, instead of using the metric $\widetilde{\mathcal{G}}_S^{\mu\nu}$, we could have used the metric $\mathcal{G}^{\mu\nu}$, the matrix inverse of $\mathcal{G}$, in the term involving $\partial_\mu\chi\partial_\nu\chi$. In order to resolve this ambiguity, we have used the result of ref. [24]. Ref. [24] analysed open string theory in the presence of constant background metric and anti-symmetric tensor field, and showed that the natural metric for open strings is the symmetric part of the inverse of $(G+B)$ where $G_{\mu\nu}$ and $B_{\mu\nu}$ are respectively the pullback of the metric and the antisymmetric tensor field on the D-brane world-volume. Since in the D-brane world-volume action the gauge field always appears in the combination $(B_{\mu\nu}+F_{\mu\nu})$, we can use the result of ref. [24] to conclude that in the presence of a constant background gauge field strength $F_{\mu\nu}$, the natural metric appearing in the kinetic term of χ is the symmetric part of the inverse of $(G+F)$. The requirement of space-time supersymmetry then fixes the form of the action (3.2).[6]

Note that although χ does not transform under supersymmetry in the gauge invariant description, it does transform under supersymmetry according to eqs. (2.16) and (2.17) in the static gauge.

3.2 Fermionic zero modes on the type I D-particle

Type I D-particle is obtained by modding out the type IIB D-particle by the world-sheet parity transformation Ω [6, 1]. The massless degrees of freedom on the world volume contains a subset of the world-volume degrees of freedom of the type IIB D-particle which are invariant under Ω, and the effective action involving these modes is given by the effective action of the type IIB D-particle with the Ω odd modes set to zero. But type I D-particle also has 32 extra fermionic zero modes from open strings stretched between the D0-brane and the space filling D9-branes which are

[6]Of course the antisymmetric part of constant background $\widetilde{\mathcal{G}}^{\mu\nu}$ has the effect of making the ordinary products in (3.2) into non-commutative $*$ products [25, 26, 27, 24]. This can be reexpressed in terms of ordinary products by including terms with higher derivatives. In the present case these extra terms vanish due to symmetry reasons.

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present in the type I string theory. Let us denote these modes by ψ^I $(0 \leq I \leq 32)$, and let us denote the world-volume time coordinate of the D0-brane by τ. We need to construct a reparametrization and supersymmetry invariant world-volume action for these modes. We propose the following action:

$$\int d\tau \psi^I \partial_\tau \psi^I \,. \tag{3.4}$$

This is manifestly supersymmetry and reparametrization invariant if we take ψ^I to be a scalar under reparametrization, and inert under supersymmetry transformation:

$$\delta_\epsilon \psi^I = 0 \,. \tag{3.5}$$

As in the previous case, ψ^I acquires a non-trivial supersymmetry transformation law in the static gauge according to the rules given in eqs. (2.16) and (2.17).

4. Effect of tachyon condensation on the non-BPS D-brane of type II string theory

The non-BPS D-brane of type II string theory in ten dimensional Minkowski space contains a tachyonic mode besides the massless modes and the infinite number of massive modes. Let us consider the effective action as a function of the tachyonic and the massless modes, obtained by integrating out all the massive modes. Since the tachyon mass^2 is of the order of the string tension, there is no systematic procedure for computing this effective action, but we shall be interested in studying some of the general properties of this action. If we ignore terms involving derivatives of $\mathcal{G}_{\mu\nu}$ and $\mathcal{F}_{\mu\nu}$, then combining the results of [24] with the requirement of supersymmetry, we can expect the following form of the effective action:

$$S = -\int d^{p+1}\sigma \sqrt{-\det(\mathcal{G}+\mathcal{F})}\, F\left(T, \partial_\mu T, D_\mu \partial_\nu T, \ldots, \widetilde{\mathcal{G}}_S^{\mu\nu}, \widetilde{\mathcal{G}}_A^{\mu\nu}\right) + I_{WZ} \,. \tag{4.1}$$

Here T is the tachyon field, and F is some function of its arguments. We have included the original action (2.5) and (2.22) into the definition of F so that for $T = 0$, $F = C$. $\widetilde{\mathcal{G}}_S^{\mu\nu}$ and $\widetilde{\mathcal{G}}_A^{\mu\nu}$ denote the symmetric and the antisymmetric parts of $\widetilde{\mathcal{G}} \equiv (\mathcal{G}+\mathcal{F})^{-1}$. The analysis of [24] tells us that the background value of $\widetilde{\mathcal{G}}_S$ gives the natural metric for the open string, whereas the effect of a background $\widetilde{\mathcal{G}}_A$ is to convert an ordinary product to a non-commutative $*$ product. I_{WZ} denotes a Wess-Zumino term representing the supersymmetric generalization of the coupling of the tachyon to background Ramond-Ramond fields [7, 2, 3, 28, 29, 30]. This term typically involves the wedge product of dT with a supersymmetry invariant p-form on the D-brane world-volume. An example of such a term will be the wedge product of dT with the Wess-Zumino term for the BPS D$(p-1)$ brane constructed in refs. [14, 15, 16].

We shall be interested in analysing this action for constant T. Thus $\partial_\mu T$ and hence I_{WZ} vanishes. For such a background the dependence of F on $\widetilde{\mathcal{G}}_S$ and $\widetilde{\mathcal{G}}_A$ disappears, since there are no indices with which $\widetilde{\mathcal{G}}_S$ can contract, and since for constant functions the $*$ product reduces to the ordinary product. Thus the action can be rewritten as:

$$-\int d^{p+1}\sigma\sqrt{-\det(\mathcal{G}+\mathcal{F})}\,V(T)\,, \tag{4.2}$$

where V is the tachyon potential. It has been argued on general grounds that at the minimum T_0 of the potential V vanishes, i.e.

$$V(T_0)=0\,. \tag{4.3}$$

From eq. (4.2) we see that at $T=T_0$ the world volume action vanishes identically. Thus in this case the world-volume gauge field acts as a lagrange multiplier field which imposes the constraint that the U(1) gauge current must vanish identically. As a result all states which are charged under the U(1) (e.g. an open string with one end on this non-BPS D-brane and the other end on another D-brane) will disappear from the spectrum.

This result is similar to the result of [19] where it was argued that this U(1) gauge field is confined at the tachyonic ground state.[7] But the mechanism proposed in [19] was non-perturbative from the point of view of the D-brane world-volume theory, whereas the mechanism discussed here is a tree level effect in the open string theory.

Since (4.2) is the key result leading to the conclusion above, let us review the origin of this equation in some detail. The main reason behind this form of the effective action for constant T is that inside the function F appearing in eq. (4.1), the indices of $\widetilde{\mathcal{G}}^{\mu\nu}_{S,A}$ always contract with the derivative factors and not with each other. This follows from the structure of open string disk amplitude. The effect of constant background $\widetilde{\mathcal{G}}$ is to modify the X^μ propagator on the boundary of the disk. Since the X^μ dependence of the tachyon vertex operator comes only from the momentum factors $e^{ik.X}$, $\widetilde{\mathcal{G}}^{\mu\nu}$ can modify only the momentum dependent factors of the correlation function of tachyon vertex operators. Thus for constant T there is no non-trivial dependence of the effective action on $\widetilde{\mathcal{G}}^{\mu\nu}$.[8] The only dependence on $\widetilde{\mathcal{G}}^{\mu\nu}$ comes through the effective coupling constant of the open string theory [24], as an overall multiplicative factor of $\sqrt{\det(\mathcal{G}+\mathcal{F})}$. Since this argument works only for constant background $\widetilde{\mathcal{G}}^{\mu\nu}$, it does not give us any information about terms involving derivatives of $\widetilde{\mathcal{G}}^{\mu\nu}$. Whether inclusion of these terms in our analysis changes the conclusion remains to be seen. Unfortunately there does not seem to be any systematic procedure for analysing these terms.

[7] Actually [19] did not look at this problem, but to a closely related problem of tachyon condensation on a brane-antibrane pair.

[8] This can also be argued using string field theory.

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